ALGORITHM FOR MODELING LOCATION SIGNALS REFLECTED FROM THE EDGE OF VARIOUS UNDERLYING SURFACES

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***Abstract* — the paper proposes an algorithm for modeling location signals reflected from the edge of underlying surfaces that differ in the reflection characteristics of location signals. The land-sea edge, the sea-ice cover, the grassy cover-forest, etc. can serve as underlying surfaces. The features of modeling such signals in the analysis of algorithms for determining the outline of the studied edges on a computer using mathematical modeling methods, in particular the shape of the coastal zone, are considered. Simulation algorithms for edge echo signals take into account not only the laws of probability distribution of reflected signals, but also their correlation and spectral characteristics.**

Keywords — modeling algorithm, location signal, land-sea edge, underlying surface, distribution law, coastal waters, spatial-correlation properties .

I. INTRODUCTION

The most common method for studying complex nonlinear systems is the method of mathematical modeling of computer systems, which is often the only method for determining the quality characteristics of the systems under study. This method is most often implemented in the form of a simulation process that allows you to determine the quality characteristics of the system in dynamics taking into account most of the conditions for its functioning.

To organize a simulation method on a computer, it is necessary to implement an algorithm for the functioning of the system under study, algorithms for simulating input information signals and processing algorithms for the output signals of the system that determine its quality characteristics in accordance with specified criteria. In this paper, we propose algorithms for modeling the input signals of small-sized airborne radars intended for mapping the earth's surface and determining the outline of the edges of underlying surfaces such as coastal outlines (land-sea edge), outlines of ice cover and free water area of "clean" water, the boundary of forest covers, and etc. In this case, of course, it is assumed that the determination of the boundary of the edges is possible only when the echo signals of the underlying surfaces differ in their statistical characteristics.

In this paper, we propose algorithms for simulating input echoes of mapping systems and determining edge contours, using aperture synthesis modes to increase accuracy. The synthesis technique is described in relation to the problem of determining the shape of the coastal zone, but the described approach to the synthesis of simulation algorithms is fully applicable to other similar problems.

II. CHARACTERISTICS OF SIGNALS REFLECTED FROM UNDERLYING SURFACES

A pulsed location signal reflected from the underlying surface at the input of the receiving device of an on-board small-sized radar is a segment of a random process observed in the range gate of the receiver. The range gate contains M range paths, the duration of each of which corresponds to the radar resolution element in range (in meters) equal to $\Delta l_r = \tau_i\, c/2$, where τ_i is the duration of the probe pulse (with a complex signal after compression), and c is the speed of light. Thus, the observed input process is represented by M samples, each of which describes a signal received from a corresponding range resolution element. If the onboard radar implements the aperture synthesis mode of the antenna device, then the signal in the range gate is the output signal of the Doppler filter, with each Doppler filter corresponding toits direction in azimuth. The filter bandwidth, depending on many radar parameters and surface observation conditions, determines the width of the resolution element in azimuth and, accordingly, its linear size (in meters) from the angular coordinate Δl_φ.

Thus, with each sounding at the output of the receiver of the onboard radar, using the aperture synthesis mode of the antenna device, we get a flat picture of the "illuminated" mapping area, presented in the form of a matrix of rows and columns, in accordance with the number of range tracks M and Doppler filters N. Therefore, when implementing the aperture synthesis mode, it is necessary to simulate these random matrices taking into account the two-dimensional spatial correlation function that describes the statistical dependences between the matrix elements.

This is already a random field modeling problem, which was theoretically solved in [1-3]. In this paper, which is actually a generalization of the materials presented in [4, 5], we restrict ourselves to a consideration of the methodology for synthesizing the random process algorithms observed in the range gate of the receiver of the onboard radar, that is, modeling algorithms for the columns of the observed matrix.

The statistical characteristics of the random process in the range strobe of the receiving device will be described by the one-dimensional amplitude distribution density and the normalized correlation function of the amplitudes, since these data obtained in experimental studies of statistical characteristics are available for different types of underlying surfaces. It should be borne in mind that a wide variety of correlation and spectral characteristics of reflections from underlying surfaces leads to the fact that the correlation functions are taken into account "roughly", namely, only the duration of the normalized correlation function at the level of 0.5 is taken into account. This makes the model used universal, but allows only the width of the fluctuation spectrum to be taken into account. For the accepted constraints, the underlying surfaces will differ in the one-dimensional laws of the distribution of amplitudes (powers) and the correlation characteristics of the sequence of amplitudes.

The methodology for the synthesis of simulation models, for concreteness, is set forth on the example of synthesis of models in determining the coastal zone, that is, for the land-sea edge. The technique is universal and does not change for other types of underlying surfaces, only the statistical characteristics of the location signals reflected from the underlying surfaces change, it is possible to refine the correlation-spectral characteristics of reflections, namely, more accurately take into account the features of correlation-spectral relationships, but this is nothing more than modification of the technique.

III. Statistical Characteristics of Sea Surface Echoes

To approximate the density distribution of the amplitudes of the signals reflected by the sea surface, the logarithmically normal distribution density, the logarithmic Weibull distribution and the K-distribution are most often used [6-12]. These three distributions do not contradict the experimental data, but the most convenient for the synthesis of echo modeling algorithms is the log-normal distribution, based on which simple and effective simulation algorithms for echo signals are synthesized that do not contain any methodological errors. Therefore, a model of a log-normal distribution was adopted in the work.

The density of the log-normal probability distribution of the amplitudes is written as follows

$$f_L(A) = \frac{1}{\sqrt{2\pi}\sigma_{LA} A} \exp\left[-\frac{\left(\ln A - \ln \bar{A}_L\right)^2}{2\sigma_{LA}^2} \right], \qquad (1)$$

where σ_{LA} and $\ln \bar{A}_L$ are the distribution parameters associated with the mathematical expectation, dispersion, and coefficient of variation by the relations [3, 5]:

$$M(A) = m_L = \bar{A}_L \cdot \exp\left(\frac{\sigma_{LA}^2}{2} \right),$$

$$D(A) = D_L = \bar{A}_L{}^2 \cdot \exp\left(\sigma_{LA}^2\right) \cdot \left(\exp\left(\sigma_{LA}^2\right) - 1 \right),$$

$$K_{LA} = \sqrt{\exp\left(\sigma_{LA}^2\right) - 1}.$$

The distribution function $F_L(A)$ for density $f_L(A)$ is equal to

$$F_L(A) = \int_0^A f_L(x) \cdot dx$$

it is not explicitly expressed in terms of elementary functions, but the calculation of the function and distribution density of a logarithmically normal value is represented by functions in any certified mathematical package, in particular, in MATLAB. Therefore, $f_L(x)$, $F_L(x)$ and the inverse distribution function $(F_L(x))^{-1}$ can be considered tabulated functions and not be involved in the derivation of their analytical representations in the form of rapidly converging series.

For the practical use of expressions, it is necessary to set the parameters σ_{LA} and $\bar{A}_L$ for a specific resolution element. These parameters are determined through the energy characteristics of the echo signals of the elements, namely, through the average power of the reflected signals $\tilde{P}_L$, determined by the well-known radar formula [12] through the area and specific effective reflective surface of the resolution element, and the coefficient of variation of the power of the echo signals K_{LP}, the values of which for different conditions for observing the sea surface and sea waves can be found in [8, 9].

The functional dependences of σ_{LA} and $\bar{A}_L$ on $\tilde{P}_L$ and K_{LP} are given in [10]

$$\sigma_{LA} = \frac{1}{2}\sqrt{\ln\left(1 + K_{LP}^2\right)},$$

$$\bar{A}_L = \frac{\sqrt{2\tilde{P}_L}}{\sqrt[4]{\ln\left(1 + K_{LP}^2\right)}}.$$

In addition to these characteristics, it is also necessary to have data on the spatial function of the correlation of echo signals $R_{LA}(\tau)$, which we will approximate by the dependence characteristic of the logarithmically normal distribution law of the form

$$R_{LA}(\tau) = \frac{1}{K_{LA}^2} \cdot \left[\left(1 + K_{LA}^2 \right)^{r_{LA}(\tau)} - 1 \right] \qquad (2)$$

where $r_{LA}(\tau)$ is the normalized correlation function of the logarithms of the amplitudes. Confining ourselves to the simplest exponential dependence, we can take as $r_{LA}(\tau)$

$$r_{LA}(\tau) = \exp(-\mu_L \cdot |\tau|) \qquad (3)$$

where μ_L is a parameter that determines the duration of the normalized correlation function $R_A(\tau)$ at a certain level. The numerical values of μ_L for τ, expressed in seconds or meters, can be found in [3, 10].

It should be noted that $R_A(\tau)$ weakly depends on the variation coefficients; therefore, expression (3) for $r_{LA}(\tau)$ can also be taken as a correlation function. Now all the necessary expressions for writing the echo-signal modeling algorithm for a separate range sea track are defined.

IV. Statistical Characteristics Of The Earth Surface Echoes

To approximate the density distribution of the amplitudes of the signals reflected by the earth's surface, we will use the Weibull distribution, which we will write, following [9], in the following form

$$f_V(A) = \alpha_{VA}\lambda_{VA}A^{\alpha_{VA}-1}\exp(-\lambda_{VA}A^{\alpha_{VA}}),$$

where the distribution parameters α_{VA} and λ_{VA} are related to the mathematical expectation m_V, dispersion D_V and coefficient of variation K_{VA} by the following dependences

$$M(A) = m_V = \frac{\lambda_{VA}^{-\frac{1}{\alpha_{VA}}},}{\alpha_{VA}}\Gamma(\frac{1}{\alpha_{VA}}),$$

$$D(A) = D_V = \lambda_{VA}^{\frac{2}{\alpha_{VA}}}\left(\frac{2}{\alpha_{VA}}\Gamma\left(\frac{2}{\alpha_{VA}}\right) - \frac{1}{\alpha_{VA}^2}\Gamma^2\left(\frac{1}{\alpha_{VA}}\right)\right),$$

$$K_V = \sqrt{2\alpha_{VA}\frac{\Gamma\left(\frac{2}{\alpha_{VA}}\right)}{\Gamma^2\left(\frac{1}{\alpha_{VA}}\right)} - 1},$$

where $\Gamma(.)$ is the gamma function.

The distribution function $F_V(A)$ for density $f_V(A)$ is equal to

$$F_V(A) = \int_0^A f_V(x)\cdot dx$$

Like $F_L(A)$, it is not explicitly expressed in terms of elementary functions, but the calculation of the Weibull function and distribution density is represented by functions in certified mathematical packages, in particular, MATLAB. Therefore, $f_V(x)$, $F_V(x)$ and the inverse distribution function $(F_V(x))^{-1}$ can be considered tabulated functions.

For the practical application of the above expressions, it is necessary, as well as for the sea surface, to specify the distribution parameters α_{VA} and λ_{VA}, for a specific resolution element. These parameters, as above, are determined through the energy characteristics of the echo signals of the elements — the average power of the reflected signals $\tilde{P}_V$ from the resolution element and the coefficient of variation of the power of the echo signals K_{VP}, the values of which for different conditions of observation of the earth's surface can be found in [8, 9, 11].

For the Weibull distribution, the power of the reflected signals also obeys the Weibull distribution, the parameters of which α_{PA} and λ_{PA}, associated with the parameters α_{VA} and λ_{VA}, expressions

$$\alpha_{PA} = \frac{\alpha_{VA}}{2},$$

$$\lambda_{PA} = \lambda_{VA}2^{\frac{\alpha_{VA}}{2}}$$

are determined from the system

$$\frac{\lambda_{VP}^{-\frac{1}{\alpha_{VP}}},}{\alpha_{PA}}\Gamma(\frac{1}{\alpha_{PA}}) = \tilde{P}_V,$$

$$2\alpha_{PA}\frac{\Gamma\left(\frac{2}{\alpha_{PA}}\right)}{\Gamma^2\left(\frac{1}{\alpha_{PA}}\right)} = 1 + K_{PV}^2.$$

This system is solved numerically, but in some special cases, it is possible to obtain an analytical solution (exponential and Rayleigh distributions).

For different types of underlying land surfaces, the parameters are given in [8, 9, 11]. As for the spatial correlation characteristics, due to the lack of reliable empirical data, they should be taken into account rudimentally during modeling - taking into account not the functional form of the normalized spatial correlation functions, but only the duration of the functions at a certain level, for example, at the level of 0.5 of the maximum, which will allow take into account the width of the spectrum of fluctuations of the echo signals of the earth's surface.

V. Algorithms for Modeling Echo Signals of a Range Track Including the Land-Sea Edge

Let the range track of the airborne radar field of view include, as noted above, M resolution elements Δl_r, numbered j = 1,2, ... M, Δl_{r1}, Δl_{r2}, ... Δl_{rM}. Let echo signals reflected by the sea surface fall into the k first elements Δl_{r1}, Δl_{r2}, ... Δl_{rk}, a part of the sea surface, whose area is γS, and a part of the earth, whose area is equal to $(1-\gamma)S$, where S is the area of the elementary resolution site, the remaining M-k-1 elements $\Delta l_{r,k+2}$, $\Delta l_{r,k+3}$,… Δl_{rM},, receive echoes of the earth's surface.

Then the simulation algorithm for the echo signals of such a range track consists of the following steps.

Step 1.

The samples of the amplitudes of the echo signals of the tracks Δl_{r1}, Δl_{r2},… Δl_{rk+1} are modeled as sequences of random variables A_{Lj}, j=1,2,…k+1, distributed according to the log-normal law (1) with the given parameters defined above and the correlation function view (2).

Algorithms for modeling a logarithmically normal sequence with a given correlation function and corresponding parameters are given in [3]. However, here we present another algorithm that uses tabulated functions $(F_L(x))^{-1}$ and $F_N(x)$ - normal distribution functions with zero mean and unit dispersion, which can be written as

$$A_{Lj} = F_L^{-1}(F_N(\eta_j)) \text{ , j = 1,2, ... k + 1}$$

where η_i is a sequence of normally distributed random variables with zero mean, unit variance and a normalized correlation function of the form (3), j = 1,2, ... k + 1.

The generated sequence will have a log-normal distribution law and a normalized correlation function of the form (2).

Step 2.

An algorithm for modeling numerical sequences distributed according to the Weibull law with a given correlation function using tabulated functions $(F_V(x))^{-1}$ and $F_N(x)$, the normal distribution function with zero mean and unit dispersion, can be written in a similar way

$$A_{Vj} = F_V^{-1}(F_N(\eta_j)) \tag{4}$$

where η_i is a sequence of normally distributed random variables with zero mean, unit variance and a normalized correlation function of the form (3), with another parameter μ_V, $j = k + 1, k + 2, \ldots M$.

Modeling of random sequences according to the presented algorithm (4) shows that the normalized correlation function of the sequence practically coincides with the sequence function η_j, but the distribution A_j obeys the Weibull distribution.

Step 3.

The echo of the resolution element, which contains reflections from the sea surface and from the earth, is now calculated. Here, as was done in [4, 5], it is necessary to form a vector sum of the reflected signals, taking into account the proportions of the “illuminated” areas of the sea and land included in the (k + 1) th resolution element.

The amplitude of the echo signal of the (k + 1) th resolution element is calculated by the expression

$$A_{LN,k+1} = \sqrt{\gamma A_{L,k+1}^2 + (1-\gamma)A_{V,k+1}^2 + 2\sqrt{\gamma(1-\gamma)}A_{L,k+1}A_{V,k+1}\cos(2\pi\varepsilon)}$$

where ξ is a random variable distributed uniformly on the interval (0, 1).

The generated sequence of amplitudes is the desired sequence of readings of the echo signals of the underlying surfaces (including the land-sea edge) observed in the range strobe of the receiving device of the on-board small-sized radar, mapping the coastal contours.

VI. Conclusion

An algorithm is proposed for modeling the distance track at the input of the receiving device of a small-sized airborne radar, mapping coastal contours. The algorithm takes into account the laws of the distribution of amplitudes of signals reflected from the underlying surfaces of the earth and sea and from the boundary of these surfaces (land-sea edges), as well as the normalized spatial correlation functions of the reflected signals. The presented algorithm uses tabulated functions of mathematical packages, which makes it universal in the implementation of modeling algorithms using certified mathematical packages. The described technique can also be used to determine the edges of other heterogeneous surfaces, while only the specified distribution laws of the amplitudes of the signals reflected by these surfaces and their spatial correlation characteristics change, and the algorithms themselves remain virtually unchanged, differing from those presented only by the inverse distribution functions of the amplitudes of the corresponding echoes -signals.

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